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TWO VARIANTS OF CALCULATION OF THE PARAMETERS OF BUBBLING PROCESSES IN AXISYMMETRIC CHANNELS WITH A VARIABLE FLOW AREA

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Consideration is given to the formulations of the primal and inverse problems of calculation of bubbling processes in axisymmetric channels with a variable flow area. A physico-mathematical model of motion of a bubble gas-liquid mixture and results of numerical calculations are presented.

In recent times, bubbling plants have found wide use in various industrial fields. Physico-chemical processes occurring when a gas is allowed to pass through the bulk of a liquid provide a basis for aeration, flotation, purification of sewage, and cleaning of a dust-laden air; they are used in the chemical industry. In a number of cases, to intensify the technological processes it becomes necessary to prescribe the required distribution of the parameters of a gas-liquid system, formed as a result of bubbling, over the height of the plant. In particular, in enrichment of minerals or biological purification of sewage, it is desirable that the gas-liquid mixture have a variable concentration of the gas phase with the higher value of it in the upper part of the bubbling plant.

As experience in the use of flotation and aeration plants with cylindrical channels shows [1], the value of the gas content in them is usually no higher than 1.5%. A substantial increase in it can be attained either by using the countercurrent scheme of motion of the liquid and gas phases [2, 3] or as a result of the application of channels with a variable flow area. In the first case the distribution of the concentrations of the phases over the height of the cylindrical channel remains constant, in practice, whereas in a gas-liquid layer taken separately one tracks their nonuniformity caused by the migration of gas bubbles from the periphery to the center [4]. Because of this, it is appropriate to consider the second approach as being the most acceptable of the indicated ones.

The use of multidimensional mathematical models to describe the dynamics and heat and mass exchange of bubble gas-liquid mixtures involves certain difficulties associated, first of all, with the current absence of an efficient method which is capable of taking into account the characteristic features of the carrier and dispersed phases and the interactions at the phase boundary. The most remarkable works (from a very short list) on this subject are devoted to solution of particular issues, namely: tracking of the paths of individual particles of the mixture [5], propagation of disturbances in the medium [6], etc. The problems in them are formulated with numerous assumptions, probably because of the desire of researchers to adjust investigation results to one numerical method of gas- and hydrodynamics or another.

One-dimensional variants modeling, for example, the rise of a single bubble in a viscous boundless fluid [7] or the cavitation and erosion of metallic surfaces because of the collapse of bubbles [8], despite the fact that they have limitations on use and are also not free from simplifying prerequisites, are distinguished by a high degree of reliability, diversity of the efficient methods of numerical implementation, and clear presentation of the basic features of the actual physical process. Thus, at least at present, they are the only efficient means for investigating in this field.

We consider the rise of a group of bubbles in a viscous fluid and investigate the influence of the variability of the cross-sectional area on the change in the gas content along the longitudinal axis of a bubbling device.

Mathematical Model. The system of equations formulated below describes a one-dimensional steady-state change, along the longitudinal axis of a bubbling channel, in the parameters of a gas-liquid system consisting of an incompressible fluid and spherical gas inclusions; the system disregards the processes of collision, fragmentation, and coagulation of dispersed particles and interphase mass exchange. The model includes the following equations:

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of conservation of the mass of the gas phase

$$\varphi_2 \rho_2 V_2 F = \varphi_{20} \rho_{20} V_{20} F_0 ;$$

of conservation of the number of dispersed inclusions

$$n V_2 F = n_0 V_{20} F_0 ; \quad n = \frac{3\varphi_2}{4\pi r^3} ;$$

of conservation of the momentum of the phases

$$\left(1 + \frac{\varphi_2}{2} + \frac{\varphi_1 \rho_1}{2\rho_2}\right) \varphi_1 \rho_1 V_1 \frac{dV_1}{dx} = -\left(\varphi_1 + \frac{\varphi_1 \rho_1}{2\rho_2}\right) \frac{d\overline{P}}{dx} - \varphi_1 n \left(f_\mu + f_r \right) - \left(1 + \frac{\varphi_2}{2} + \frac{\varphi_1 \rho_1}{2\rho_2}\right) \varphi_1 \rho_1 g ,$$

$$\left(1 + \frac{\varphi_2}{2} + \frac{\varphi_1 \rho_1}{2\rho_2}\right) \varphi_2 \rho_2 V_2 \frac{dV_2}{dx} = -1.5\varphi_2 \frac{d\overline{P}}{dx} + \varphi_1 n \left(f_\mu + f_r \right) - \left(1 + \frac{\varphi_2}{2} + \frac{\varphi_1 \rho_1}{2\rho_2}\right) \varphi_2 \rho_2 g ,$$

$$f_\mu = \frac{1}{2} C_\mu \pi r^2 \rho_1 \left| V_1 - V_2 \right| (V_1 - V_2) , \quad f_r = 2\pi r^2 \rho_1 V_1 \Theta (V_1 - V_2) , \quad \overline{P} = \varphi_1 P_1 + \varphi_2 (P_2 - 2\Sigma / r) ;$$

$$C_\mu = \begin{cases} \dfrac{24 \left(1 + \frac{1}{6} \mathrm{Re}^{1/3}\right)}{\mathrm{Re}\,(1 - \varphi_2)} & \text{for } \mathrm{Re} < 15 \text{ [9]} , \\[2ex] \dfrac{68}{\mathrm{Re}\,(1 - \varphi_2)} & \text{for } 15 \le \mathrm{Re} \le 500 , \\[2ex] \dfrac{\frac{1845}{\mathrm{Re}} + \frac{192}{\sqrt{\mathrm{Re}}} + 5.07}{1 - \varphi_2} & \text{for } \mathrm{Re} > 500 \text{ [7]} ; \end{cases}$$

of conservation of the energy of the gas phase

$$\varphi_2 \rho_2 V_2 c_V \frac{dT_2}{dx} = \frac{\varphi_2 P_2 V_2}{\rho_2} \frac{d\rho_2}{dx} + nq , \quad q = 4\pi\alpha (T_1 - T_2) r^2 = 2\pi r \,\mathrm{Nu}\, \lambda (T_1 - T_2) ;$$

of radial deformation of bubbles

$$(1 - \psi^{(1)}) r V_2 \left(\Theta \frac{dV_2}{dx} + V_2 \frac{d\Theta}{dx}\right) = \frac{P_2 - P_1 - 2\Sigma / r}{\rho_1} - \frac{4\mu_1 V_2 \Theta}{r \rho_1} - 1.5 (1 - \psi^{(2)}) V_2^2 \Theta^2 + (1 - \psi^{(3)}) \frac{V_1 - V_2}{4} ,$$

$$\psi^{(1)} \approx \frac{1.1\varphi_2^{1/3} - \varphi_2}{\varphi_1} , \quad \psi^{(2)} \approx \frac{1.5\varphi_2^{1/3} - 1.3\varphi_2}{\varphi_1} , \quad \psi^{(3)} \approx \frac{\varphi_2}{\varphi_1} \quad \text{[9]} ;$$

for the volume concentrations of the components of the phases

$$\varphi_1 + \varphi_2 = 1 ;$$

for the pressure distribution of the fluid

$$P_1 = P_{1h} + \rho_1 g (H - x) ;$$

of state of the gas

$$P_2 = \rho_2 R T_2 ;$$

for interphase heat exchange [7]

$$\mathrm{Nu} = 1.3\,\mathrm{Pr}^{0.15} + 0.66\,\mathrm{Re}^{0.5}\,\mathrm{Pr}^{0.31} .$$

For numerical integration we must reduce the ordinary differential equations of the system to the canonical form

$$\frac{d\Phi}{dx} = f_k ,$$

where $\Phi = \{V_1, V_2, r, T_2, \Theta\}$. The content of the right-hand sides of these equations is largely determined by the formulation of the problem. It is appropriate to single out here two classes of problems: the primal problem and the inverse one.

Primal Problem. The parameters of the gas-liquid mixture are calculated in the case of its motion in an axisymmetric channel of prescribed shape. The basic calculated parameters will be as follows: gas content φ_2, velocity V_2 and radius r of the bubbles located in a gas-liquid layer which is perpendicular to the axis of symmetry of the channel and is considered separately, temperature of the gas inside the bubbles T_2, and velocity of the liquid in the layer V_1. The last two parameters can be disregarded in the cases of equality of the liquid and gas temperatures.

In addition to the equations presented, we must introduce into consideration the relation describing the change in the cross-sectional area of the channel as a function of its height. It is convenient to put it into the form

$$F = F_0 + \frac{F_h - F_0}{H} x + bx (H - x) + cx (H^2 - x^2) ,$$

where b and c are the constants characterizing the curvilinearity of the generatrix of the channel wall. According to the analogous expression, we can prescribe the distribution of the liquid temperature over the height.

The reduced ordinary differential equations were solved by the Runge–Kutta method as applied to the system air–water. On the basis of the conditions of stability of the method, the variable value of the integration step was taken in accordance with the expression

$$s = \frac{0.001}{\max\limits_k \{f_k\}} ,$$

where f_k (k = 1, 2, 3, 4, 5) is the value of the right-hand side of the kth ordinary differential equation written in canonical form with allowance for $\Theta = dr/dx$. The initial conditions required for integration were used in the following form: $V_1 = V_{10}$, $V_2 = V_{20}$, $T_2 = T_{20}$, $r = r_0$, $\Theta = 0$, $n = n_0$, $\varphi_{20} = \frac{4}{3}\pi r_0^3 n_0$, $\varphi_{10} = 1 - \varphi_{20}$, $P_{10} = P_{1h} + \rho_1 g H$, $P_{20} = P_{10} + \frac{2\Sigma}{r_0}$, and $\rho_{20} = \frac{P_{20}}{RT_{20}}$.

Results of the calculation of a change in the gas content over the height in cylindrical (curves 1 and 3) and conical (curves 2 and 4) channels are presented in Fig. 1. The initial portion of rise is characterized by a sharp decrease in the gas content of the layers, located here, because of the increase in the velocity of rise of the bubbles upon separation from the openings of gas-intake devices, which leads to a decrease in their number in a separately considered gas-liquid layer.

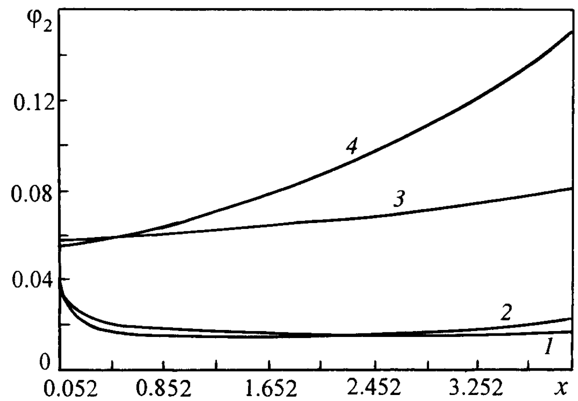

Fig. 1. Dependences of the change in the gas content over the channel height: 1) F = const and $V_{10} \neq 0$; 2) $F \neq$ const and $V_{10} \neq 0$; 3) F = const and $V_{10} \neq 0$; 4) $F \neq$ const and $V_{10} = 0$. x, m; F, m^2; V_{10}, m/sec.

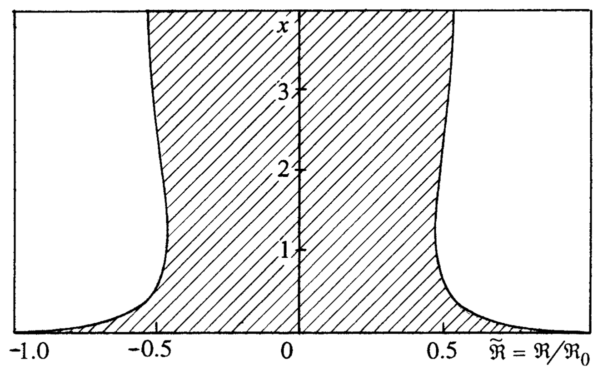

Fig. 2. Configuration of a bubbling channel ensuring the required gas-content distribution. x, m; $\mathfrak{R}$ and $\mathfrak{R}_0$, m.

The increase in the bubble-rise velocity is especially enhanced by the presence of a cocurrent liquid flow (curves 1 and 2). The character of the arrangement of the dependences in Fig. 1 yields that a substantial increase in the gas content over the height can be attained in bubbling the gas through a layer of immobile liquid in a convergent channel (curve 4).

Inverse Problem. A distinctive feature of this approach is calculation of the parameters of the gas-liquid system simultaneously with determination of the cross-sectional area of the channel at each iteration step (which gives an idea of the configuration of the bubbling channel as a whole), whereas the distribution of the gas content over the height is prescribed for one reason or another. The case considered here is of greatest practical interest since it enables one to attain the optimum regimes of the processes and to decrease the overall dimensions of the plants used. For further numerical implementation of the model it is more convenient to prescribe the dependence for the gas content in the form of a third-degree polynomial:

$$\varphi_2 = \sum_{k}^{4} A_k x^{k-1} ,$$

where A_k (k = 1, 2, 3, 4) are the prescribed numerical factors.

Figure 2 shows as an example the calculated shape of the channel that ensures, through its length, the most favorable medium for the vital functions of microorganisms constituting activated sludge in dispersing the air through the sewage. The distribution of the gas content is prescribed by the expression

$$\varphi_2 = 0.00672x^3 - 0.0691x^2 + 0.3117x + 0.045 .$$

The configuration given in Fig. 2 is characterized by the accumulation of gas bubbles in the lower part of the channel due to the substantial decrease in the cross-sectional area, the formation of a nearly foamy structure of the gas-liquid mixture in the central part with an invariable area, and the decrease in the concentration of the liquid because of its flowing out under gravity in a slightly divergent upper region.

NOTATION

C_μ, coefficient of resistance of the disperse particle to flow; c_p, specific heat of the liquid at constant pressure; c_v, specific heat of the gas at constant volume; F, cross-sectional area of the channel; f_μ and f_r, interphase forces per disperse particle (respectively, resistance to flow and inertialess component of the additional-mass force); g, acceleration of gravity; H, linear dimension over the height; n, number of disperse inclusions in a unit volume of the gas-liquid system; P, pressure; $\overline{P}$, averaged pressure in the volume of the gas-liquid mixture; q, heat flux; R, gas constant; $\mathfrak{R}$, channel radius; r, bubble radius; T, temperature; V, velocity; x, coordinate reckoned from the dispersion device; α, heat-transfer coefficient; λ, thermal conductivity; μ, coefficient of dynamic viscosity; ρ, density; Σ, surface-tension coefficient; φ_i, volume concentration of the ith phase (i = 1, 2); $\psi^{(1)}$, $\psi^{(2)}$, and $\psi^{(3)}$, coefficients taking into account the influence of the lack of singleness of bubbles on the value of the additional masses; $\mathrm{Nu} = 2r\alpha/\lambda$, Nusselt number; $\mathrm{Pr} = c_p \mu/\lambda$, Prandtl number; $\mathrm{Re} = 2r\left|V_1 - V_2\right| \rho_1/\mu$, Reynolds number; $\Theta = dr/dx$, convective component of the velocity of the interphase boundary referred to the velocity of a bubble; s, integration step used for the axis of the bubbling device. Subscripts: 1, carrier phase; 2, dispersed phase; 0, initial value of the parameter; h, value of the parameter on the free liquid surface; k, ordinal number of the element of a massif.

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